

NUMBER BASES

Summary:

1. Number bases are different ways of writing down numbers.
2. The most common base system is base 10.
3. The digits of a number in any base are less than the base itself
4. The digits 10 and 11 are represented by **t** and **e** respectively in number bases

NOTE:

(i) Base 10 is called **decimal base**

(ii) Base 2 is called **binary base**

(iii) Base 3 is called **trinary base**

(iv) Base 8 is called **octal base**

EXAMPLES:

1. Convert the following to base ten

(i) 1011_{two}

(ii) 346_{seven}

(iii) 2210_{three}

(iv) $2et_{\text{twelve}}$

(v) $312 \cdot 21_{\text{four}}$

(vi) $0 \cdot 12_{\text{six}}$

solution

$$\begin{aligned} \text{(i)} \quad 1011_{\text{two}} &= (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \\ &= (1 \times 8) + (0 \times 4) + (1 \times 2) + (1 \times 1) \\ &= 11_{\text{ten}} \end{aligned}$$

$$\begin{aligned}
 \text{(iv) } 312 \cdot 21_{\text{four}} &= (3 \times 4^2) + (1 \times 4^1) + (2 \times 4^0) + (2 \times 4^{-1}) + (1 \times 4^{-2}) \\
 &= (3 \times 16) + (1 \times 4) + (2 \times 1) + (2 \times \frac{1}{4}) + (2 \times \frac{1}{16}) \\
 &= 54 + \frac{1}{2} + \frac{1}{16} \\
 &= 54 \frac{9}{16}_{\text{ten}} \quad \text{or} \quad 54 \cdot 5625_{\text{ten}}
 \end{aligned}$$

CONVERTING FROM BASE TEN TO OTHER BASES

Summary:

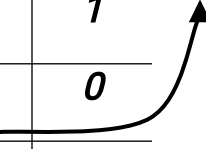
(i) Divide the number repeatedly by the required bases

(ii) The remainder in reverse order gives the required number

EXAMPLES:

1. Convert 64_{ten} to base three

3	64	R
3	21	1
3	7	0
	2	1



$$\therefore 64_{\text{ten}} = 2101_{\text{three}}$$

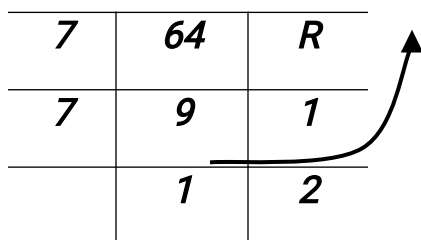
2. Convert 246_{ten} to base five

3. Convert 2101_{three} to base seven

Hint: First convert 2101_{three} to base ten

$$\begin{aligned} 2101_{\text{three}} &= (2 \times 3^3) + (1 \times 3^2) + (0 \times 3^1) + (1 \times 3^0) \\ &= 64_{\text{ten}} \end{aligned}$$

7	64	R
7	9	1
	1	2



$$\therefore 2101_{\text{three}} = 121_{\text{seven}}$$

5. Find the value of n in the following equations:

$$(i) 45_n = 1112_{\text{three}} \quad (ii) 21_n = 19_{\text{ten}} \quad (iii) 303_n = 410_{\text{six}}$$

$$(iv) 202_n = 37_{\text{nine}} \quad (v) 112_n + 304_n = 421_n$$

OPERATIONS WITH ANY BASE OTHER THAN 10

ADDITION:

If the sum of the digits exceeds the base, divide that sum by the base then write down the remainder and carry the whole number.

EXAMPLES:

1. Workout the following leaving your answer in the base indicated

$$(i) 136_{\text{seven}} + 254_{\text{seven}}$$

$$(ii) \ 232_{\text{five}} + 344_{\text{five}}$$

$$(iii) \ 28 \cdot 57_{\text{nine}} + 6 \cdot 34_{\text{nine}}$$

Solution:

$$(i) \begin{array}{r} 136_{\text{seven}} \\ + 254_{\text{seven}} \\ \hline 423_{\text{seven}} \end{array}$$

$$(ii) \begin{array}{r} 232_{\text{five}} \\ + 344_{\text{five}} \\ \hline 1131_{\text{five}} \end{array}$$

$$(iii) \begin{array}{r} 28 \cdot 57_{\text{nine}} \\ + 6 \cdot 34_{\text{nine}} \\ \hline 36 \cdot 02_{\text{nine}} \end{array}$$

2. Workout $122_{\text{three}} + 461_{\text{seven}}$ giving your answer in base five

Hint: First convert 122_{three} and 461_{seven} to base ten and then finally express the answer in the required base

$$122_{\text{three}} = (1 \times 3^2) + (2 \times 3^1) + (2 \times 3^0) = 17_{\text{ten}}$$

$$461_{\text{seven}} = (4 \times 7^2) + (6 \times 7^1) + (1 \times 7^0) = 239_{\text{ten}}$$

$$\Rightarrow 122_{\text{three}} + 461_{\text{seven}} = 17_{\text{ten}} + 239_{\text{ten}} = 256_{\text{ten}}$$

5	256	R
5	51	1
5	10	1
	2	0

$$\therefore 122_{\text{three}} + 461_{\text{seven}} = 2011_{\text{five}}$$

SUBTRACTION:

In case of borrowing the new value is the sum of the base and the digit which was small.

EXAMPLES:

1. Workout the following leaving your answer in the base indicated

(i) $72_{\text{eight}} - 43_{\text{eight}}$

(ii) $254_{\text{eight}} - 217_{\text{eight}}$

(iii) $30 \cdot 241_{\text{five}} - 14 \cdot 143_{\text{five}}$

Solution:

(i)
$$\begin{array}{r} 72_{\text{eight}} \\ - 43_{\text{eight}} \\ \hline 27_{\text{eight}} \end{array}$$

$$\begin{array}{r}
 (ii) \quad 254_{\text{eight}} \\
 - 217_{\text{eight}} \\
 \hline
 35_{\text{eight}}
 \end{array}$$

$$\begin{array}{r}
 (iii) \quad 30 \cdot 241_{\text{five}} \\
 + 14 \cdot 143_{\text{five}} \\
 \hline
 14 \cdot 043_{\text{five}}
 \end{array}$$

2. Workout $221_{\text{three}} - 101_{\text{two}}$ giving your answer in base four

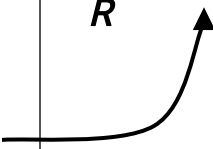
Hint: First convert 221_{three} and 101_{two} to base ten and then finally express the answer in the required base

$$221_{\text{three}} = (2 \times 3^2) + (2 \times 3^1) + (1 \times 3^0) = 25_{\text{ten}}$$

$$101_{\text{two}} = (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = 5_{\text{ten}}$$

$$\Rightarrow 221_{\text{three}} - 101_{\text{two}} = 25_{\text{ten}} - 5_{\text{ten}} = 20_{\text{ten}}$$

	20	R
4		
4	4	0
	1	1



$$\therefore 221_{\text{three}} - 101_{\text{two}} = 110_{\text{four}}$$

MULTIPLICATION AND DIVISION

EXAMPLES:

1. Workout the following leaving your answer in the base indicated

(i) $152_{\text{eight}} \times 43_{\text{eight}}$

(ii) $et5_{\text{twelve}} \times 8t_{\text{twelve}}$

(iii) $124_{\text{five}} \times 32_{\text{five}}$

Solution:

(i) 152_{eight}

$$\begin{array}{r} \times 43_{\text{eight}} \\ \hline 476 \\ + 650 \\ \hline 7176_{\text{eight}} \end{array}$$

(ii) $et5_{\text{twelve}}$

$$\begin{array}{r} \times 8t_{\text{twelve}} \\ \hline 9t82 \\ + 7te4 \\ \hline 88t02_{\text{twelve}} \end{array}$$

$$\begin{array}{r}
 \text{(iii)} \quad 124_{\text{five}} \\
 \times 32_{\text{five}} \\
 \hline
 303 \\
 + 432 \\
 \hline
 10123_{\text{five}}
 \end{array}$$

2. Workout $1011_{\text{two}} \times 12_{\text{three}}$ giving your answer in binary base

Hint: First convert 1011_{two} and 12_{three} to base ten and then finally express the answer in the required base

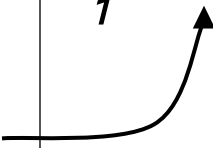
$$1011_{\text{two}} = (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) = 11_{\text{ten}}$$

$$12_{\text{three}} = (1 \times 3^1) + (2 \times 3^0) = 5_{\text{ten}}$$

$$\Rightarrow 1011_{\text{two}} \times 12_{\text{three}} = 11_{\text{ten}} \times 5_{\text{ten}} = 55_{\text{ten}}$$

2	55	R
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2	27	1
2	13	1
	6	1
2		
2	3	0
	1	1



$$\therefore 1011_{\text{two}} \times 12_{\text{three}} = 110111_{\text{two}}$$

3. Workout the following leaving your answer in the base indicated

(i) $2001_{\text{three}} \div 12_{\text{three}}$

(ii) $110111_{\text{two}} \div 101_{\text{two}}$

Solution:

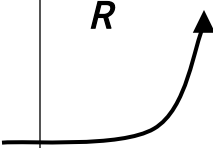
(i) *Hint: First convert 2001_{three} and 12_{three} to base ten and then finally express the answer in the required base*

$$2001_{\text{three}} = (2 \times 3^3) + (0 \times 3^2) + (0 \times 3^1) + (1 \times 3^0) = 55_{\text{ten}}$$

$$12_{\text{three}} = (1 \times 3^1) + (2 \times 3^0) = 5_{\text{ten}}$$

$$\Rightarrow 2001_{\text{three}} \div 12_{\text{three}} = 55_{\text{ten}} \div 5_{\text{ten}} = 11_{\text{ten}}$$

	11	R
3		
3	3	2
	1	0



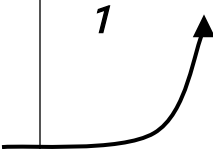
$$\therefore 2001_{\text{three}} \div 12_{\text{three}} = 102_{\text{three}}$$

$$\begin{aligned} \text{(ii) } 110111_{\text{two}} &= (1 \times 2^5) + (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \\ &= 55_{\text{ten}} \end{aligned}$$

$$101_{\text{two}} = (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = 5_{\text{ten}}$$

$$\Rightarrow 110111_{\text{two}} \div 101_{\text{two}} = 55_{\text{ten}} \div 5_{\text{ten}} = 11_{\text{ten}}$$

2	11	R
	5	1
2		
2	2	1
	1	0



$$\therefore 110111_{\text{two}} \div 101_{\text{two}} = 1011_{\text{two}}$$

EER:

1. Convert the following to base ten

(i) 2212_{three}

(ii) 1011_{two}

(iii) 234_{five}

2. Express 0.24_{six} as a fraction in base ten

3. Express 45.3_{six} in base ten using point notation

4. Find the value of n if $45_n = 100001_{\text{two}}$

5. Find the value of n if $103_n + 26_n = 131_n$

6. Convert 102_{three} to binary base

7. Workout the following leaving your answer in the base indicated

(i) $152_{\text{eight}} \times 43_{\text{eight}}$

(ii) $et5_{\text{twelve}} \times 8t_{\text{twelve}}$

(iii) $2212_{\text{three}} \div 21_{\text{three}}$

8. Arrange the following numbers 36_{eight} , 302_{four} and 202_{three} in ascending order

